Materials
Notes

- No lecture next Friday.
- Home work to be handed out next Thursday.
- Hand in date 21st November, 4pm
Poisson’s ratio

\[ \nu = - \frac{\Delta d / d_0}{\Delta L / L_0} \]

Material constant \( \approx 0.3 \)

\[ \nu = - \frac{\varepsilon_x}{\varepsilon_z} = - \frac{\varepsilon_y}{\varepsilon_z} \]
A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm. Determine the magnitude of the load required to produce a $2.5 \times 10^{-3}$ mm change in diameter if the deformation is entirely elastic.

When the force $F$ is applied, the specimen will elongate in the $z$ direction and at the same time experience a reduction in diameter, $\Delta d$, of $2.5 \times 10^{-3}$ mm in the $x$ direction. For the strain in the $x$ direction,

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3} \text{ mm}}{10 \text{ mm}} = -2.5 \times 10^{-4}$$

which is negative, since the diameter is reduced.

It next becomes necessary to calculate the strain in the $z$ direction using

The value for Poisson’s ratio for brass is 0.34, and thus

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{(-2.5 \times 10^{-4})}{0.34} = 7.35 \times 10^{-4}$$
Example

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The applied stress may now be computed

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^3 \text{ MPa}) = 71.3 \text{ MPa}$$

$$F = \sigma A_0 = \sigma \left( \frac{d_0}{2} \right)^2 \pi$$

$$= (71.3 \times 10^6 \text{ N/m}^2) \left( \frac{10 \times 10^{-3} \text{ m}}{2} \right)^2 \pi = 5600 \text{ N (1293 lbf)}$$
THE END OF ELASTIC DEFORMATION

Strain Offset Method

Nearly all engineering applications ensure that the materials used undergo elastic deformation only.
Resilience is the capacity of a material to absorb energy under elastic deformation and release it upon reloading in such a way as to recover the energy. This is associated with the modulus of resilience $U_r$ which is the strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding.

$$U_r = \int_0^{\varepsilon_y} \sigma \, d\varepsilon$$
This is the realm of non recoverable mechanical response, i.e permanent deformation.
Breaking Stuff

Tensile Strength

Yield Strength

Plastics Limit

Young Modulus

Fracture
Question

Calculate the Young's Modulus

The Yield strength at offset 0.002

The maximum load that can be sustained by a cylindrical specimen diam. 12.8 mm

Change in length of 200 mm specimen under stress of 345 MPa
Definitions

**Tensile Strength** [after yielding the stress necessary to continue plastic deformation increases at a maximum the TENSILE STRENGTH]. This is the maximum stress the system can take. This is not useful in engineering as you want to work in the plastic region. If your system is at its TENSILE STRENGTH it is well beyond its design.

**Yield Strength** [The stress at which the system has passed the plastic limit, this can be defined in a number of ways, generally it is defined as a strain offset.]
Some more definitions

Hooke’s Law

\[ \sigma = E \varepsilon \]

Hooke’s Law for shear

\[ \tau = G \gamma \]  \hspace{1cm} \text{Shear strain}

\[ \text{Shear stress} \quad \text{Shear Modulus} \]

The shear and \textbf{elastic} modulus can be related by the Poissons ratio

\[ G = \frac{E}{2(1+\nu)} \]
A few more

For larger strains the Compressive (K) modulus and the Young’s modulus can be related

\[ K = \frac{E}{3(1-2\nu)} \]

\[ E = \frac{9GK}{3K+G} \]
What happens at the Yield point

Atoms rearrange themselves into lower energy configurations

Two methods are outlined here

Slip and twinning

Twining occurs when a whole plane rearranges to reduce stress concentration
Slipping

This simple explanation does not give us a particularly good answer

(a) Slip plane

(b) Shear stress as a function of relative displacement of the planes from their equilibrium positions.
Defects

Figure 5.11 shows how the preferred slip planes are oriented relative to the crystal structure. It is important to note that the three vectors in the figure are not the same. It is also important to note that the three vectors in Figure 5.12 are not the same. It is also important to note that the three vectors in Figure 5.12 are not the same.

Shear stress

Stress

Strain

Upper yield point

Lower yield point

Shear stress

Slip plane

Edge dislocation line

Unit step of slip

(c)

Figure 5.11 (a) Shear stress, (b) Shear stress, (c) Shear stress.
All materials undergo elastic extension to some point, what happens after that is defined by the materials atomic structure.

**Ductility** (AKA plastic flow)

\[ \% \text{EL} = \left( \frac{l_f - l_0}{l_0} \right) \times 100 \]

\[ \% \text{RA} = \left( \frac{A_0 - A_f}{A_0} \right) \times 100 \]
Elastic recovery

![Graph showing elastic recovery](image)
Flexural Strength

Possible cross sections

Support

\[ F \]

\[ L \]

\[ b \]

\[ d \]

Rectangular

Circular

\[ \sigma = \text{stress} = \frac{Mc}{I} \]

where \( M \) = maximum bending moment
\( c \) = distance from center of specimen to outer fibers
\( I \) = moment of inertia of cross section
\( F \) = applied load

Rectangular

<table>
<thead>
<tr>
<th>( M )</th>
<th>( c )</th>
<th>( I )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{FL}{4} )</td>
<td>( \frac{d}{2} )</td>
<td>( \frac{bd^3}{12} )</td>
<td>( \frac{3FL}{2bd^2} )</td>
</tr>
</tbody>
</table>

Circular

<table>
<thead>
<tr>
<th>( M )</th>
<th>( c )</th>
<th>( I )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{FL}{4} )</td>
<td>( R )</td>
<td>( \frac{\pi R^4}{4} )</td>
<td>( \frac{FL}{\pi R^3} )</td>
</tr>
</tbody>
</table>

\[ \sigma_{fs} = \frac{3LF_f}{2bd^2} \]

\[ \sigma_{fs} = \frac{LF_f}{\pi r^2} \]

Used to study materials that are very brittle or not at all malleable, generally ceramics.
Toughness is a mechanical term that is used in several contexts; loosely speaking, it is a measure of the ability of a material to absorb energy up to fracture. Specimen geometry as well as the manner of load application are important in toughness determinations. For dynamic (high strain rate) loading conditions and when a notch (or point of stress concentration) is present, notch toughness is assessed by using an impact test, as discussed in Section 9.8. Furthermore, fracture toughness is a property indicative of a material’s resistance to fracture when a crack is present (Section 9.5).
# Modeling True Stress

\[ \sigma_T = K \varepsilon_T^n \]

<table>
<thead>
<tr>
<th>Material</th>
<th>( n )</th>
<th>MPa</th>
<th>psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-carbon steel (annealed)</td>
<td>0.26</td>
<td>530</td>
<td>77,000</td>
</tr>
<tr>
<td>Alloy steel (Type 4340, annealed)</td>
<td>0.15</td>
<td>640</td>
<td>93,000</td>
</tr>
<tr>
<td>Stainless steel (Type 304, annealed)</td>
<td>0.45</td>
<td>1275</td>
<td>185,000</td>
</tr>
<tr>
<td>Aluminum (annealed)</td>
<td>0.20</td>
<td>180</td>
<td>26,000</td>
</tr>
<tr>
<td>Aluminum alloy (Type 2024, heat treated)</td>
<td>0.16</td>
<td>690</td>
<td>100,000</td>
</tr>
<tr>
<td>Copper (annealed)</td>
<td>0.54</td>
<td>315</td>
<td>46,000</td>
</tr>
<tr>
<td>Brass (70Cu–30Zn, annealed)</td>
<td>0.49</td>
<td>895</td>
<td>130,000</td>
</tr>
</tbody>
</table>

Effect on temp (polymer)

Hardness tests are performed more frequently than any other mechanical test for several reasons:

1. They are simple and inexpensive—ordinarily no special specimen need be prepared, and the testing apparatus is relatively inexpensive.

2. The test is nondestructive—the specimen is neither fractured nor excessively deformed; a small indentation is the only deformation.

3. Other mechanical properties often may be estimated from hardness data, such as tensile strength.